Aircraft Runway Length Estimation

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Organization of this Section

- Understanding basic aircraft weights and performance limits
- General Equations of Motion to Understand Runway Length Curves and Tables
- General Federal Aviation Regulation Criteria to Develop Runway Length Requirements at Airports
- General Methods to Estimate Runway Length at Airports

Understanding Aircraft Weights and its Limits

Aircraft Mass Definitions

There are several important aircraft operational characteristics to know about the aircraft mass. Aircraft mass expenditures are significant and thus need to be accounted for in the air vehicle runway length analysis.

OEW

operating empty weight (or mass) is the weight (or mass) of the aircraft without fuel and payload (just the pilots and empty seats)

MTOW

maximum takeoff operating weight (or mass) - structurally the maximum demonstrated mass at takeoff for safe flight

MALW

maximum allowable landing weight (or mass) is the maximum demonstrated landing weight (or mass) to keep the landing gear intact at maximum sink rate (vertical speed)

MSPW

maximum structural payload weight (or mass) is the maximum demonstrated payload to be carried without stressing the aircraft fuselage

MZFW

maximum zero fuel weight (or mass) is the sum of the OEW and the MSPW

MTW

maximum taxi weight (or mass) of the maximum demonstrated weight (or mass) for ground maneuvering. Usually slightly more than MTOW

All aircraft operating weight limits are established during the certification of the vehicle (FAR part 25 - for transport aircraft or FAR 23 for smaller aircraft)

Operational Definitions

DTW	desired takeoff weight (or mass) is the weight of the aircraft considering fuel (includes reserve), payload and OEW to complete a given stage length (trip distance)

$$DTW = PYL + OEW + FW$$

PYL	is the payload carried (passengers and cargo)
OEW	is the operating empty weight
FW	is the fuel weight to be carried (usually includes reserve fuel)

Runway Length Estimation Procedures

Factors influencing runway length performance

- Performance requirements imposed by FAR regulations (such as FAR 25, 23, 121, 91, etc.)
- Environmental characteristics (temperature and pressure) of the airport in question
  Operating limits on aircraft weight

Methods to calculate runway length

- AC 150/5325-4 (tables and graphs)
- Use of aircraft manufacturer data (such as Boeing data)
- Declared distance concept (AC 150/5300-13 Appendix 14)

General Equations of Motion to Understand Runway Length Curves and Tables

Introductory Remarks on Aircraft Performance

Air vehicles are significant different than their ground vehicle counterparts in three aspects:

- Aircraft require a prepared surface to lift-off and fly which affects the overall capability of the vehicle to carry useful payload
- Aircraft move in a dynamic atmospheric environment where changes in temperature, density, and speed of sound are substantial and cannot be neglected
- Aircraft mass expenditures are significant and thus need to be accounted for in the air vehicle performance analysis. For example, a Boeing 747-400 can takeoff at near 390 metric tons and yet land at its destination at 220 metric tons thus making the fuel expenditure a significant factor in how the vehicle performs along the flight path.

International Standard Atmosphere (1976)

Characteristics of the International Standard Atmosphere (ISA)

Geopotential Altitude (m.)	Temperature (^{o}K) T	Density (kg/m^3) ρ	Speed of Sound (m/s) a
0	288.2	1.225	340.3
1000	281.7	1.112	336.4
2000	275.2	1.007	332.5
3000	268.7	0.909	328.6
4000	262.2	0.819	324.6
5000	255.7	0.736	320.5
6000	249.2	0.660	316.4
7000	242.7	0.589	312.3
8000	236.2	0.525	308.1
9000	229.7	0.466	303.8
10000	223.2	0.413	299.5
11000	216.7	0.364	295.1
12000	216.7	0.311	295.1
13000	216.7	0.266	295.1
14000	216.7	0.227	295.1
15000	216.7	0.194	295.1

The density in Denver Colorado Is 15% lower than sea level

The density in Mexico City Is 20% lower than sea level

Lower density affects the performance of aircraft engines (less thrust produced)

Bottom line: Aircraft need more runway for takeoff and landing

Interactive International Atmosphere Model available at: https://www.digitaldutch.com/atmoscalc/

Important Aircraft Speed Terms to Know

Indicated Airspeed (IAS)

is the speed registered in the cockpit instrument

True Airspeed (TAS)

is the actual speed of the vehicle with respect of the mass of air surrounding the aircraft (accounts for compressibility effects)

Calibrated Airspeed (CAS)

similar to IAS but corrected for instrument position errors (airflow problems outside the vehicle).

Ground Speed (GS)

TAS corrected for wind

Stalling Speed (V_{stall})

minimum speed for safe flight

Mach Number

ratio of the aircraft speed to the speed of sound, a (note a varies with altitude)

Aircraft Runway Length Performance Estimation

Critical issue in airport engineering and planning (the wrong runway length is costly to the operator and perhaps unsafe).

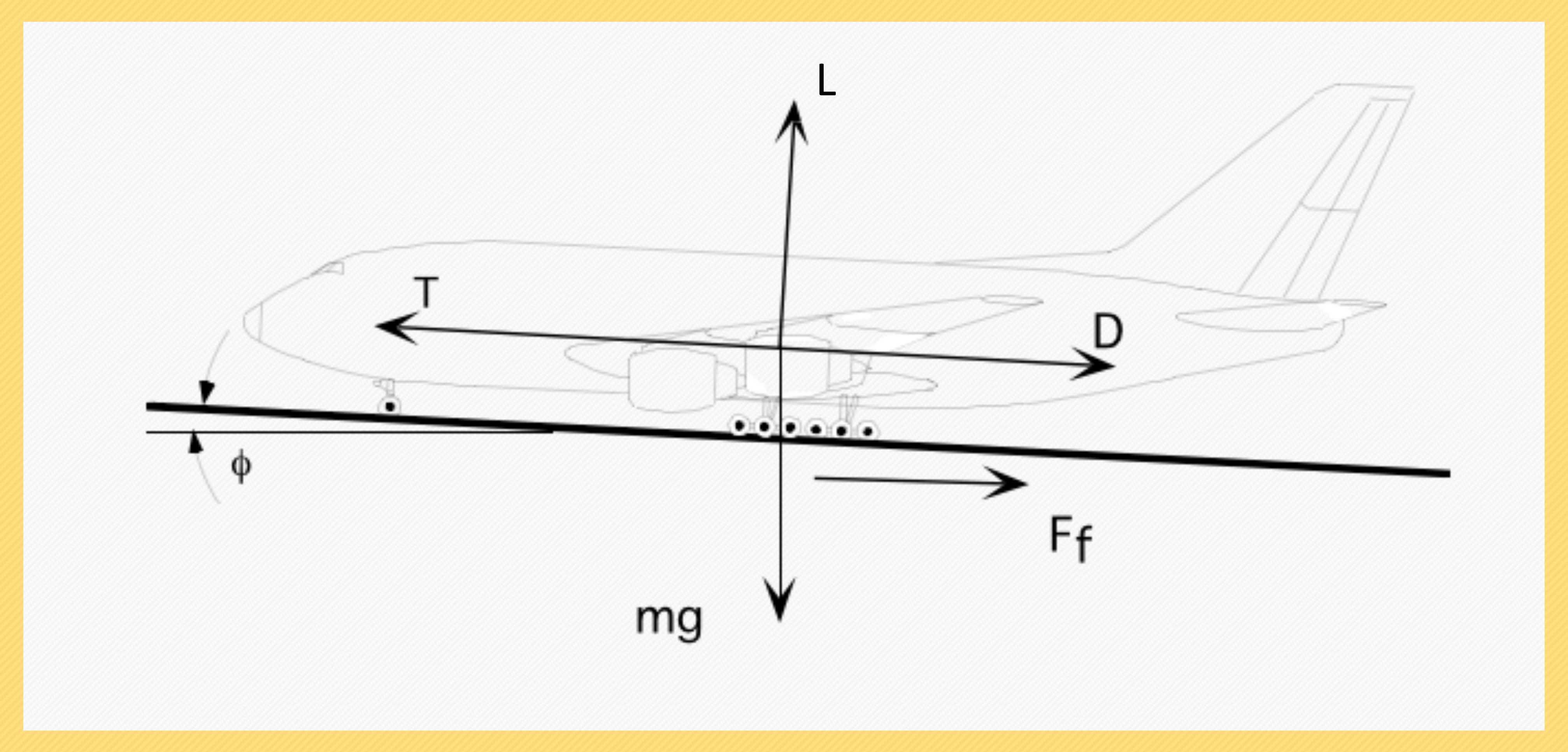


Figure 2.1 Forces acting in the aircraft during takeoff

Nomenclature

T	L	D	F_f
thrust force (also called tractive effort) provided by the vehicle powerplant	lifting force provided by the wing-body of the vehicle	drag force to the vehicle body, nacelle(s), landing gears, etc.,	friction force due to rolling resistance

The functional form of these forces has been derived from dimensional analysis (review your math course notes) and from extensive knowledge of fluid mechanics (wind tunnels and water tank experiments)

Functional Forms of the Forces

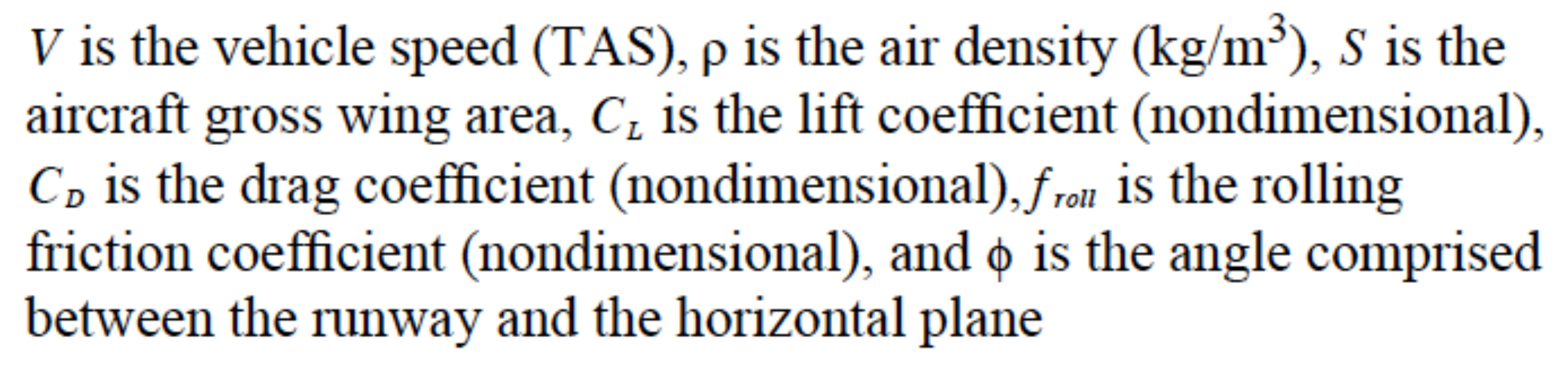
The functional form of these forces is as follows:

$$L = \frac{1}{2}\rho V^{2} S C_{L} \tag{2.3}$$

$$D = \frac{1}{2}\rho V^{2} S C_{D} \tag{2.4}$$

$$T = f\ (V, \rho) \tag{2.5}$$

$$F_{f} = (mg \cos \phi - L)\, f_{roll} \tag{2.6}$$



V is the vehicle speed (TAS), ρ is the air density (kg/m^3), S is the aircraft gross wing area, C_L is the lift coefficient (nondimensional), C_D is the drag coefficient (nondimensional), f_{roll} is the rolling friction coefficient (nondimensional), and ϕ is the angle comprised between the runway and the horizontal plane

Notes on Various Parameters

1) C_L and C_D are specific to each airframe-flap configuration

2) f_{roll} is usually a function of runway surface conditions and aircraft speed

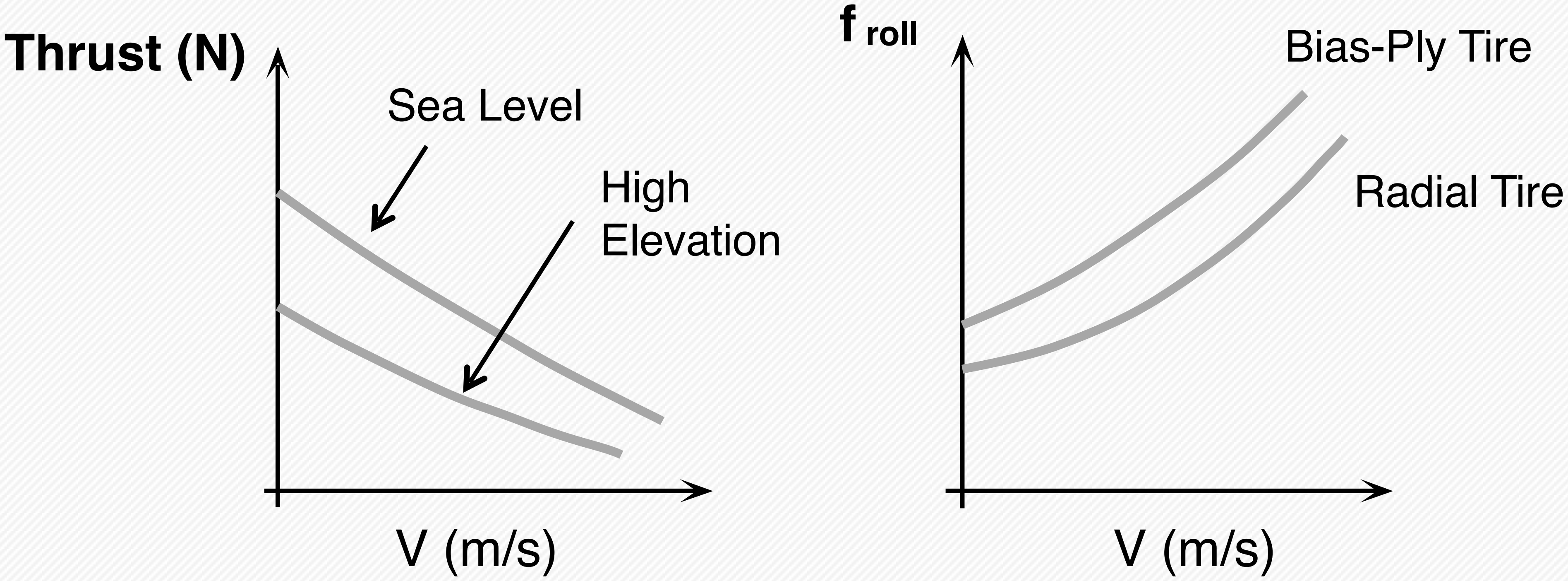


Figure 2.2. Typical Variations of *Thrust (T)* and f_{roll} with Aircraft Speed.

Estimating Runway Acceleration

Using Newton's second law and summing forces in the horizontal direction of motion (*x*),

$$ma_x = T(V, \rho) - D - (mg\cos\phi - L)\, f_{roll} - mg\sin\phi \tag{2.7}$$

Variations of *T* (tractive effort or thrust) and f_{roll} can be assumed to be linear with respect to airspeed for the range of speed values encountered in practice. For small angles this equation can be expressed as,

$$ma_x = T(V, \rho) - D - (mg - L)\, f_{roll} \tag{2.8}$$

$$ma_x = T(V, \rho) - \frac{1}{2}\rho V^2 sc_D - \left(mg - \frac{1}{2}\rho V^2 SC_L\right) f_{roll} \tag{2.9}$$

$$\boxed{a_x = \frac{1}{m}\Big(T(V, \rho) + \frac{1}{2}\rho V^2 S(C_L f_{roll} - C_D) - mg\, f_{roll}} \tag{2.10}$$

Remarks About Equation 2.10

- The acceleration capability of the aircraft decreases as speed is gained during the takeoff roll due to a reduction in the thrust produced by the engines

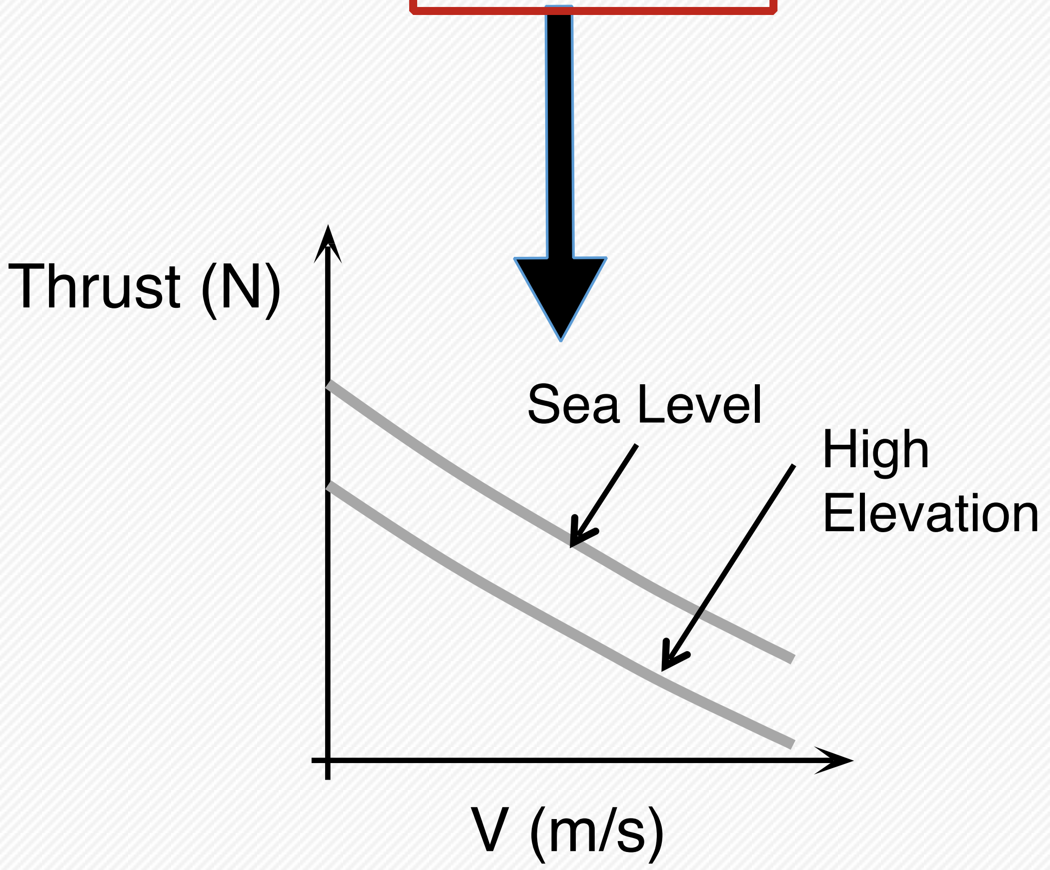
- If Eq. 2.10 is integrated twice between an initial speed, V_0 and the lift-off speed, V_{lo} , the distance traversed during the takeoff roll can be found

- Usually this requires a computer simulation since many parameters such as T and f_{roll} vary with speed (time varying) making the coefficient of the differential equation of motion time dependent.

Remarks About Equation 2.10

- The acceleration of the aircraft decreases as speed is gained during the takeoff roll due to a **reduction** in the **thrust produced** by the engines
- Takeoff runway length increases if acceleration is reduced

$$a_x = \frac{1}{m}\boxed{(T(V, \rho)} + \frac{1}{2}\rho V^2 S(C_L f_{roll} - C_D) - mg f_{roll}$$

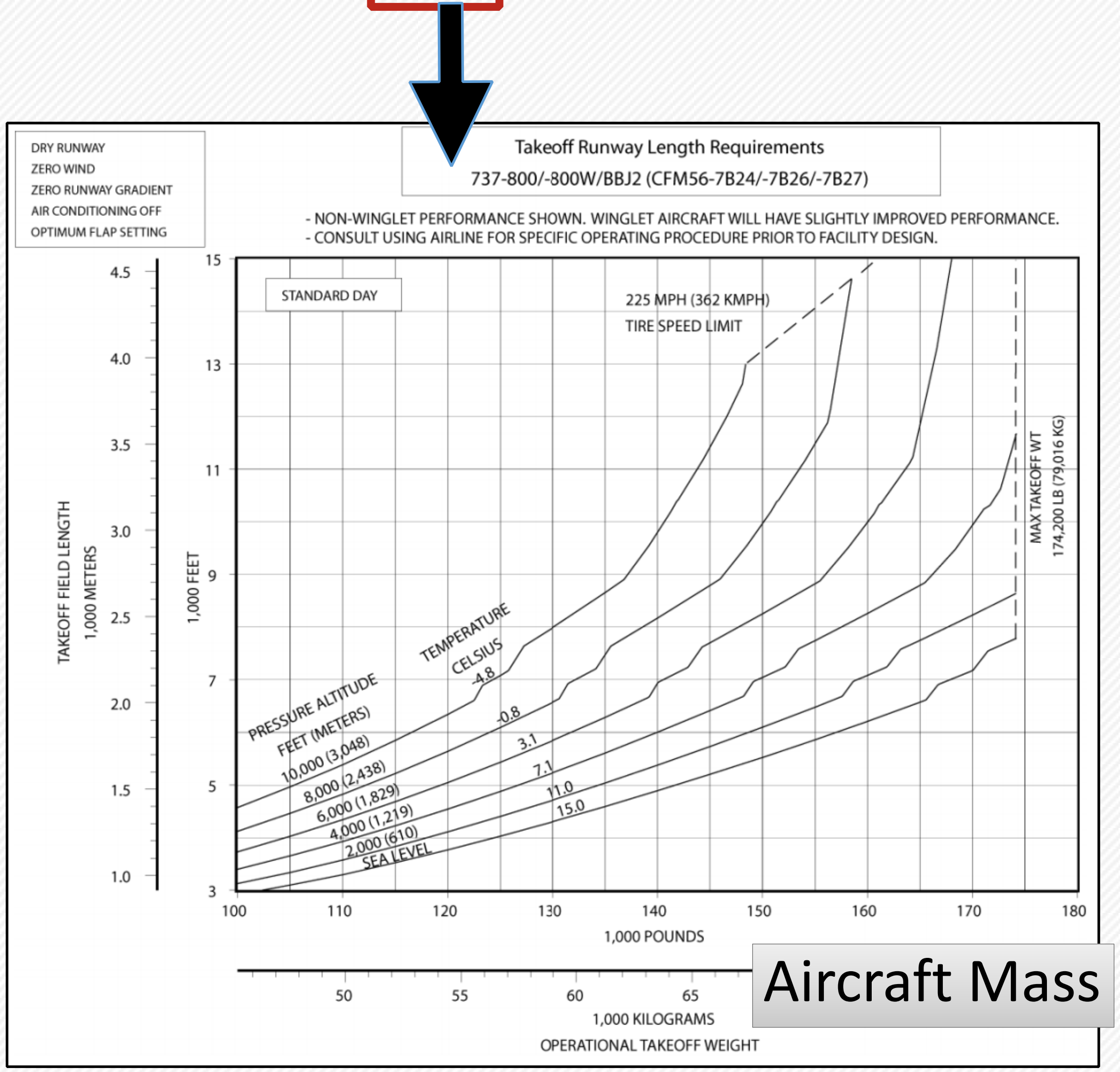


- Aircraft acceleration decreases at high elevation airports due to a reduction in the thrust produced by the engines (lower density reduces the mass flow rate of air into the engine)

Remarks About Equation 2.10

- Aircraft acceleration decreases when aircraft mass increases
- Takeoff runway length increases if acceleration is reduced

$$a_x = \boxed{\frac{1}{m}}\left(T(V,\rho) + \frac{1}{2}\rho V^2 S\left(C_L f_{roll} - C_D\right) - mg f_{roll}\right.$$



- Note the nonlinear effect of takeoff runway length at high elevation airfields
- At high elevation airports, the reduced air density requires aircraft to reach higher ground speeds for a safe takeoff

Effect of Aerodynamic Coefficients

The flap setting affects C_D and C_L and hence affects acceleration and runway length required for a takeoff. Typical variations of C_D with flap angle are shown below

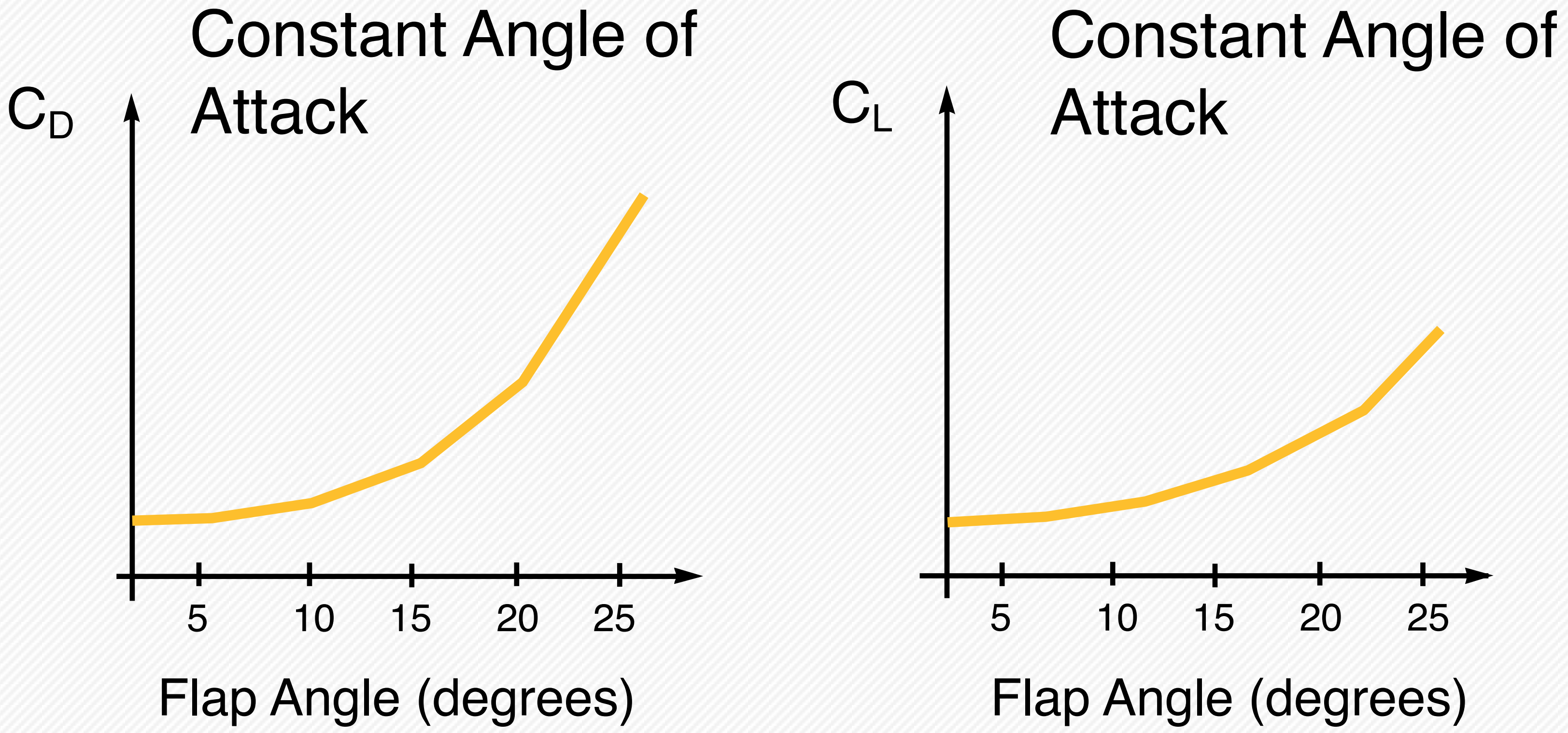
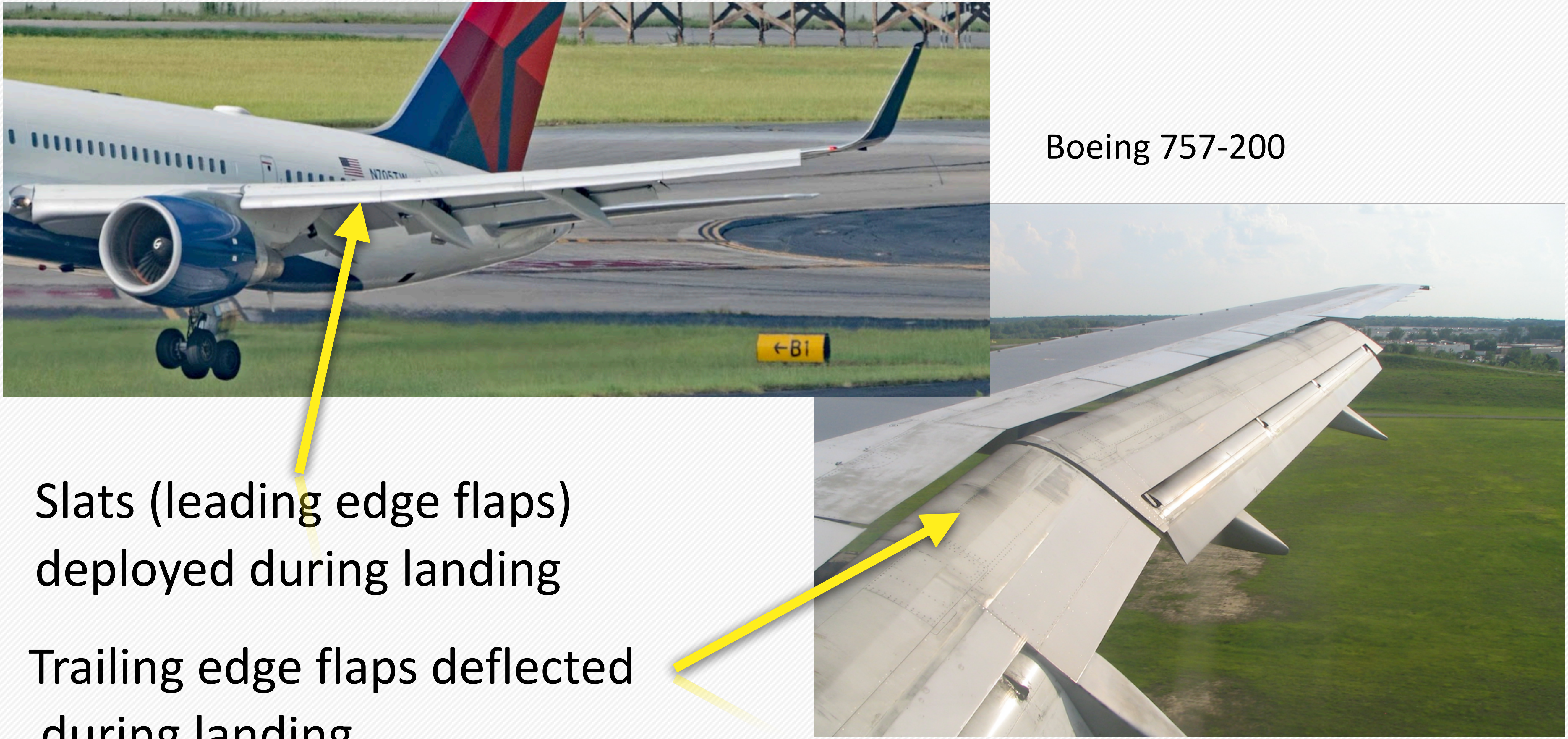


Figure 2.3. Typical Variations of C_D and C_L with Aircraft Wing Flap Angle.

Flaps and Slats (Leading Edge Flaps)

1) C_L and C_D are specific to each airframe-flap configuration

2) f_{roll} is usually a function of runway surface condition and aircraft speed



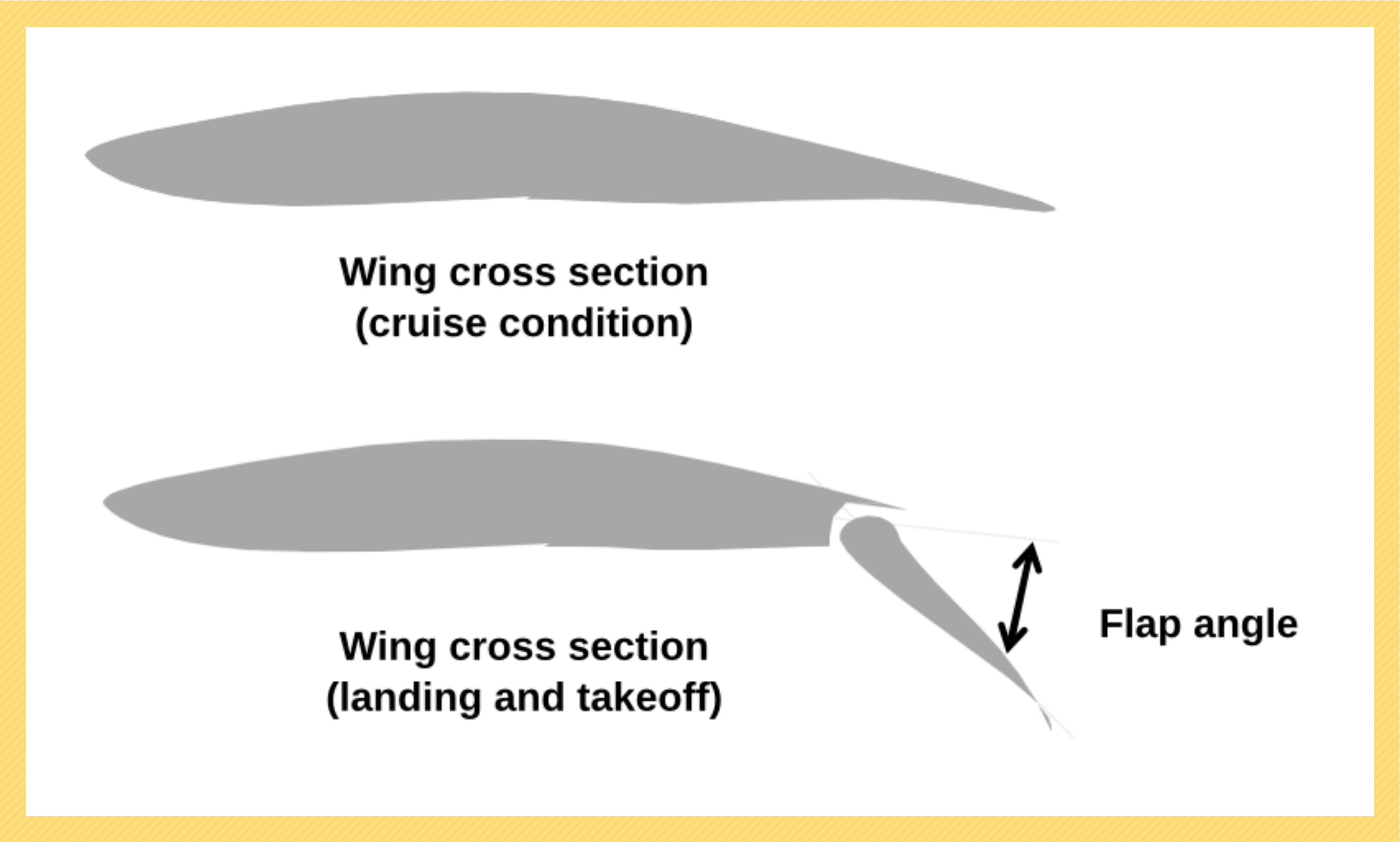
Boeing 757-200

Slats (leading edge flaps) deployed during landing

Trailing edge flaps deflected during landing

Flap Angle

- Angle formed between the flap chord and the wing chord
- Flaps are used to increase lift (but they increase drag too) during takeoff and landing
- Flaps reduce the stalling speed of the aircraft



Split flap of the Douglas DC-3

Examples

Airbus A321neo
Takeoff

Leading edge flaps

Trailing edge double slotted flaps (10 degrees)

Airbus A321neo
Landing

Leading edge flaps

Trailing edge double slotted flaps (30 degrees)

Example Flap Settings for Commercial Aircraft

Aircraft	Takeoff Flap Settings (degrees) or Label	Landing Flap Settings (degrees) or Label
Boeing 737-900	0,1, 5, 10,15, 20	25, 30, 40
Airbus A320	18/10 (1+F)	22/15 (2), 22/20 (3), 27/35 (Full)
Boeing 787-8	0,1,5,15, 20	25, 30
Boeing 757-200	0, 1, 5, 15	25, 30
Airbus A330-300	1+F	2, Full
Airbus A350-900	1+F, 2	2, Full
Boeing 777-300ER	0,1, 5, 15, 20	25, 30

Airbus uses labels for slats (leading edge) /flaps (training edge) settings independently

Remarks About Aerodynamic Coefficients

- An increase in flap angle increases both C_L and C_D. However, these increments are not linear

- Increasing the flap angle (δ_f) increases C_L and thus reduces the lift-off speed required for takeoff due to an increase in the lifting force generated (See Equation 2.3 and replace V_1 by V).

- Increments in flap angle increases the value of C_D more rapidly which reduces the acceleration of the aircraft on the runway and increases the runway length necessary to reach the lift off speed

Final Remarks

The mass of the aircraft affects its acceleration (according to Newton's second law).

- Larger takeoff masses produce corresponding increments in the runway length requirement.

The density of the air, ρ decreases with altitude

- Lower thrust generation at high airfield elevations
- The runway length increases as the airport elevation increases
- The density also affects the second and third terms in Equation 2.10 (less drag at higher altitude)

Aircraft Operational Practices (Takeoff)

At **small flap settings** (i.e., 5 or 10 degrees) the takeoff runway length is increased due to small gains in C_L (little increase in the lifting force). Useful for high-hot takeoff conditions.

At **medium flap angle settings** (15-25 degrees) the gains in lift usually override those of the drag force. These are the flap settings typically used for takeoff except under extremely abnormal airport environments such as high elevation, hot temperature airport conditions and high aircraft weights or a combination of both. Note that the **maximum allowable takeoff weight (MTOW)** increases as the takeoff flap setting is reduced.

At **large flap angles** ($>$ 25 degrees) C_D **is excessive** and the airplane requires large takeoff runway lengths. These flap settings are only used for landing since pilots want to land at the lowest speed possible thus reducing runway length.

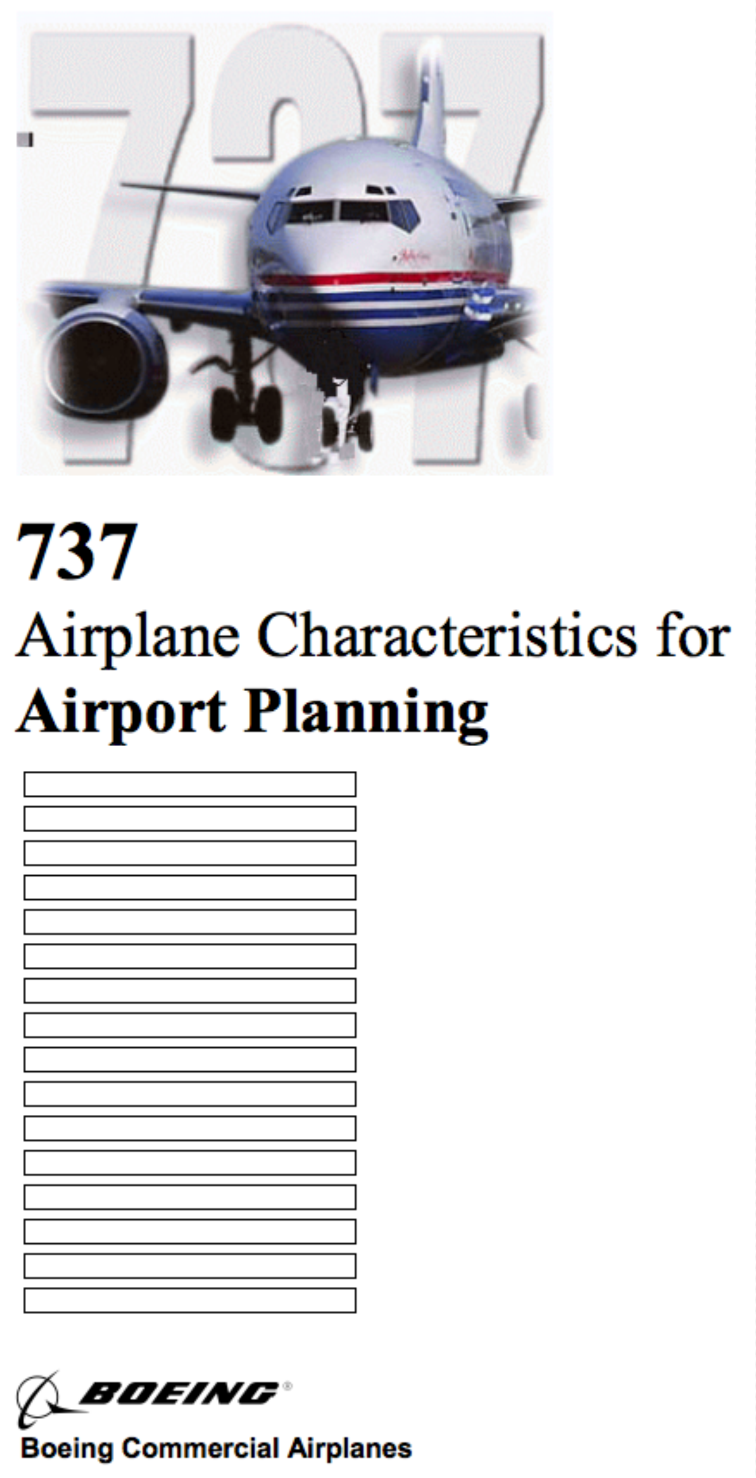
Application of Equations of Motion to Takeoff Runway Length Requirements

Equation 2.9 describes the motion of an air vehicle as it accelerates on a runway from an initial speed V_o to a final liftoff speed $V_{lof.}$

This equation can be theoretically integrated twice with respect to time to obtain the distance traveled from a starting point to the point of liftoff.

With a little more effort we could also predict the distance required to clear a 35 ft. obstacle as required by Federal Aviation Regulations Part 25 that sets airworthiness criteria for aircraft in the U.S. Airport engineers use tabular or graphical data derived from this integration procedure.

Practical Example for Commercial Transport



737

Airplane Characteristics for

Airport Planning

Boeing Commercial Airplanes

- Consider the effects of airport elevation in the runway performance of a Boeing 737-800 aircraft
- Engines are GE/Snecma CFM56-7B24/-7B26/-7B27 producing 26,000 lb of thrust
- See Boeing document D6-58325-6 at: http://www.boeing.com/boeing/commercial/airports/plan_manuals.page

Identify the Aircraft

Boeing 737-800

Twin engine commercial airliner. Two emergency exits over the wing. Most versions have winglets (shown)

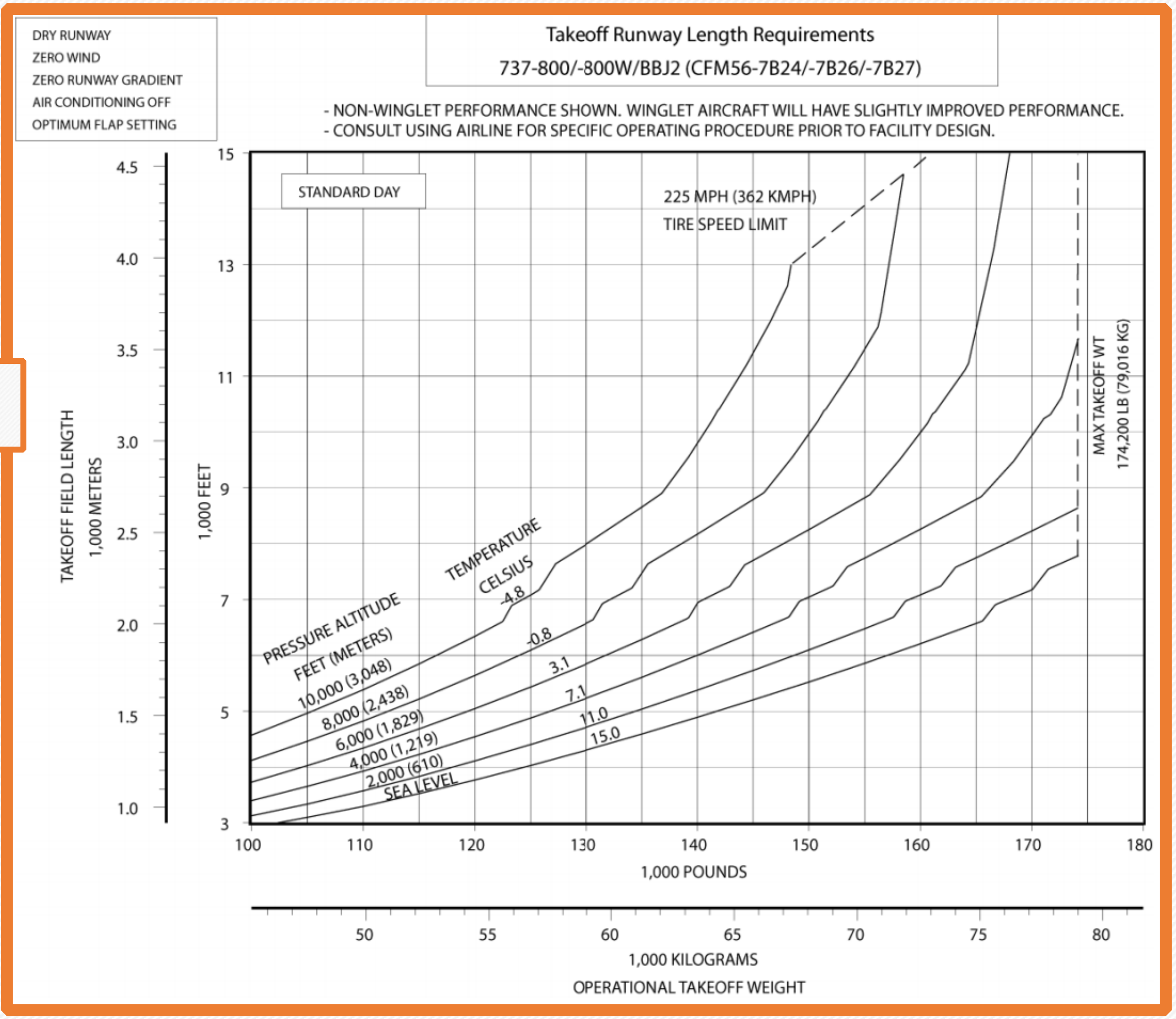
Sample Performance Chart - Boeing 737-800

Takeoff Runway Length ISA Conditions. Source: Boeing (2011)

Numerical integration of fundamental equation of motion produces the graph on the right

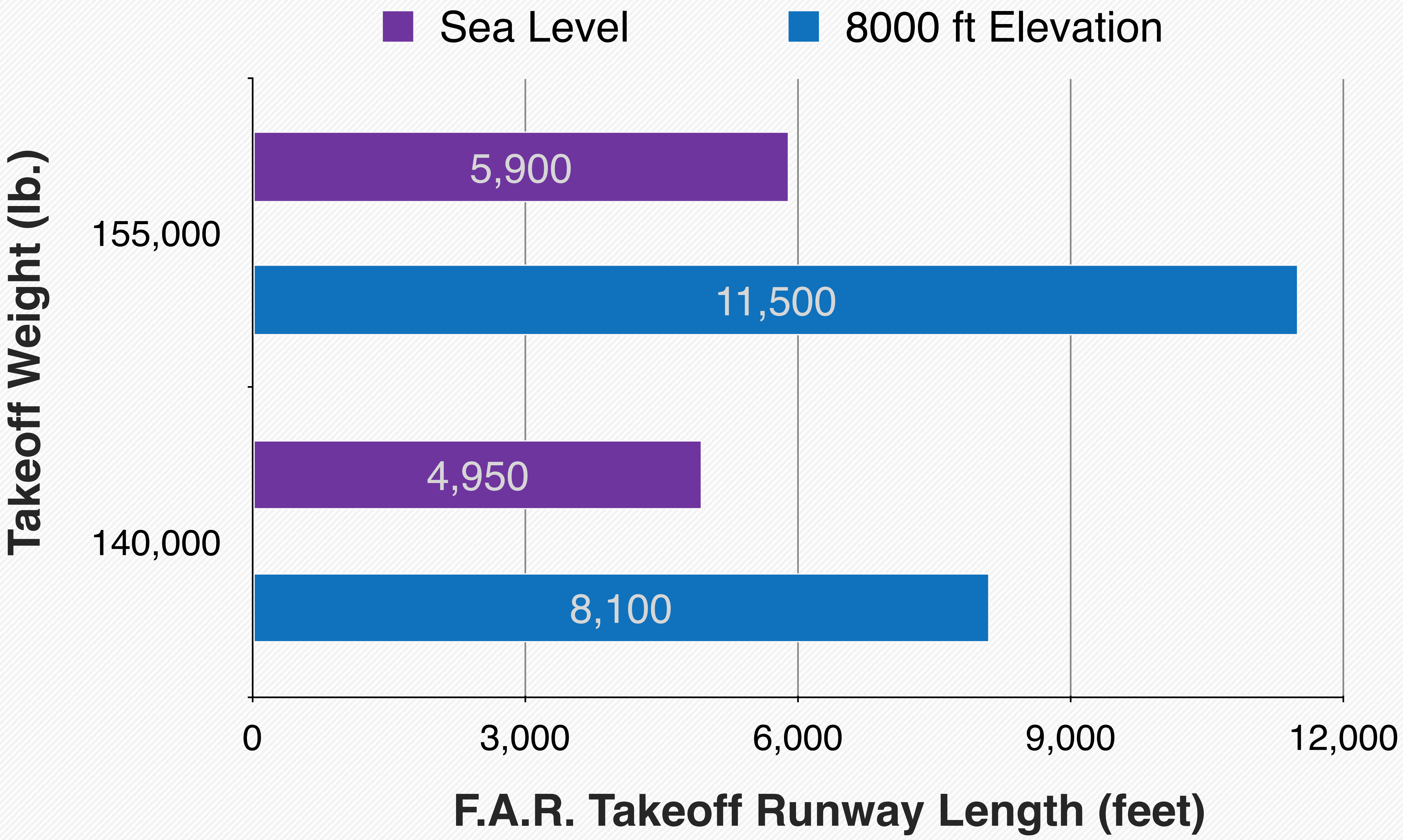
$$a_x = \frac{1}{m}(T(V,\rho) + \frac{1}{2}\rho V^2 S(C_L f_{roll} - C_D) - mg f_{roll}$$

Aircraft manufacturers perform extensive flight testing of the aircraft performance during the certification of the aircraft



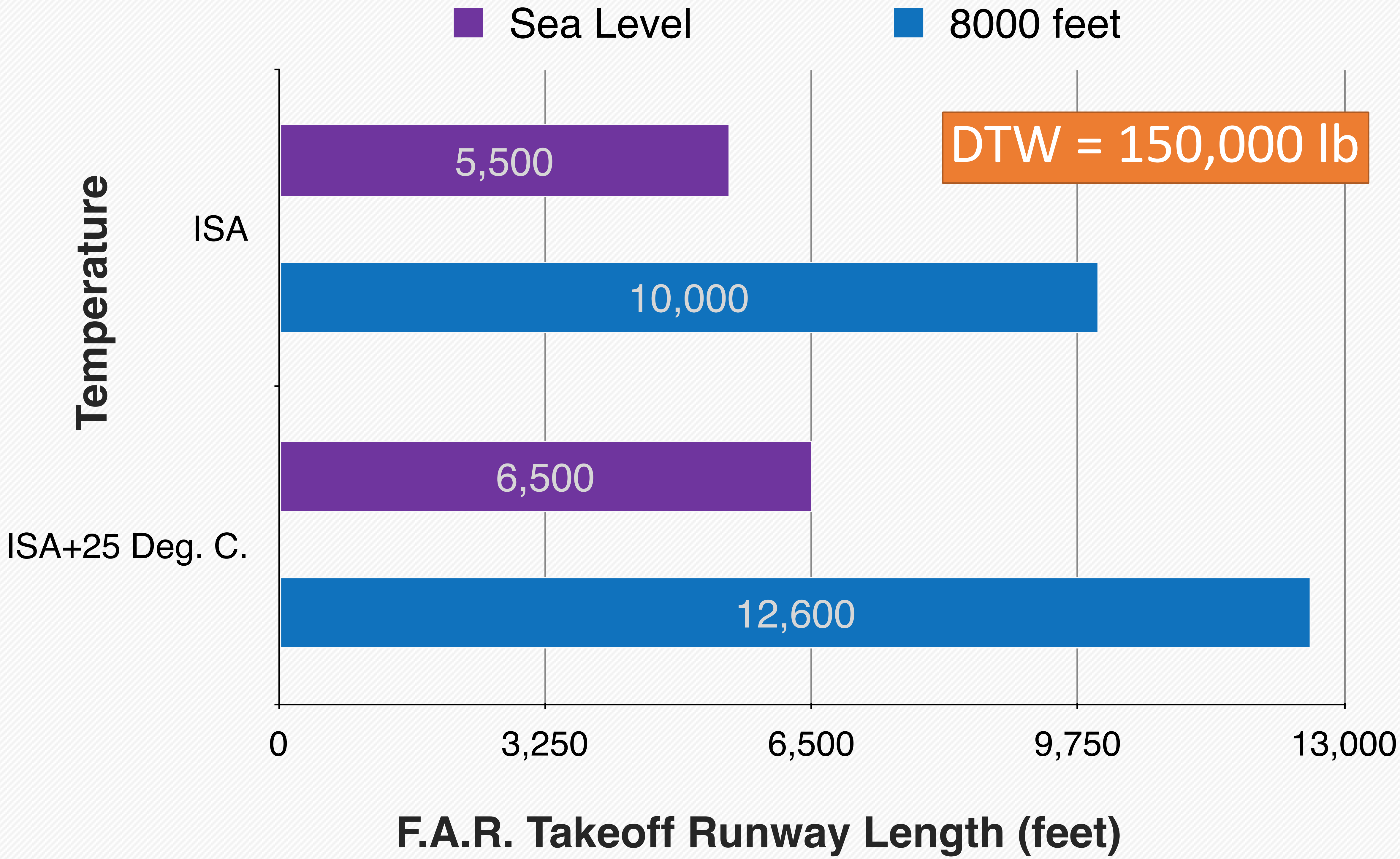
Practical Example Boeing 737-800

Takeoff Runway length with airfield elevation and aircraft weight



Practical Example Boeing 737-800

International Standard Atmosphere (ISA) conditions (see ISA table)



Observed Trends

Airfield Elevation Effect

A Boeing 737-800 requires 94% more runway departing from an airport located 8000 feet above sea level than an airport at sea level with a typical weight of 155,000 lb. (MTOW is 172,500 lb.)

Temperature Effects

The Boeing 737-800 requires 26% more runway departing from a high elevation airfield (i.e., 8,000 ft) when the temperature increases by 25 deg. C.

The increase in F.A.R. runway length is 18% when departing an airport at sea level conditions.

Federal Aviation Regulations (FAR)

Regulations (FAR Part 25) specify that:

- Aircraft should lift off at 10% above the stalling speed (V_{lof})
- Aircraft climb initially at 20% above the stalling speed (V_2)
- Aircraft speed during a regular approach be 30% above the stalling speed (V_{app})
- During takeoff aircraft should clear an imaginary 11 m (35 ft.) obstacle
- During landing aircraft should cross the runway threshold 15 m (50 ft.) above ground)

All assumptions are used to estimate takeoff and landing distances.

https://www.ecfr.gov/current/title-14/chapter-I/subchapter-C/part-25?toc=1

Explanations : Stalling and and Lift-off Speeds

The stalling speed can be estimated from the basic lift equation

$$L = \frac{1}{2} \rho V^2 S C_L$$

Under steady flight conditions $L \cong mg$ so,

$$V = \sqrt{\frac{2mg}{\rho S C_L}}$$

define C_{Lmax} as the maximum attainable (before stall) lift coefficient , then

$$V_{stall} = \sqrt{\frac{2mg}{\rho S C_{Lmax}}}$$

Sample Calculations (sea level)

Parameters of a typical Narrow Body aircraft (Boeing 737-800)

Boeing 737-800 liftoff at ATL runway 26L

$S = 125$	Square meters
$\rho = 1.225$	kg/m^3 (sea level)
$m = 75000$	kg
$C_{lmax} = 1.8$	Dimensionless
$g = 9.81$	m/s^2

$$V_{stall} = \sqrt{(2 * 75000 * 9.81)/(1.225 * 125 * 1.8)}$$

$$V_{stall} = 73.07\ m/s = 142\ knots$$

$$V_{stall} = \sqrt{\frac{2mg}{\rho S C_{Lmax}}}$$

Calculate the lift-off and climb speeds:

Liftoff speed = 1.1 V_{stall}

Climb speed = 1.2 V_{stall}

$$V_{liftoff} = 73.07 * 1.1 = 80.37\ m/s = 156\ knots$$

$$V_{climb} = 73.07 * 1.2 = 87.68\ m/s = 170\ knots$$

Calculations (10,000-foot Airport)

Parameters of a typical Narrow Body aircraft (Boeing 737-800)

Boeing 737-800 liftoff at ATL runway 26L

$S = 125$	Square meters
$\rho = 0.9046$	kg/m^3 (sea level)
$m = 75000$	kg
$C_{lmax} = 1.8$	Dimensionless
$g = 9.81$	m/s^2

$$V_{stall} = \sqrt{(2 * 75000 * 9.81)/(0.9046 * 125 * 1.8)}$$

$$V_{stall} = 85.03\ m/s = 165\ knots$$

$$V_{stall} = \sqrt{\frac{2mg}{\rho S C_{Lmax}}}$$

Calculate the lift-off and climb speeds:

Liftoff speed = 1.1 V_{stall}

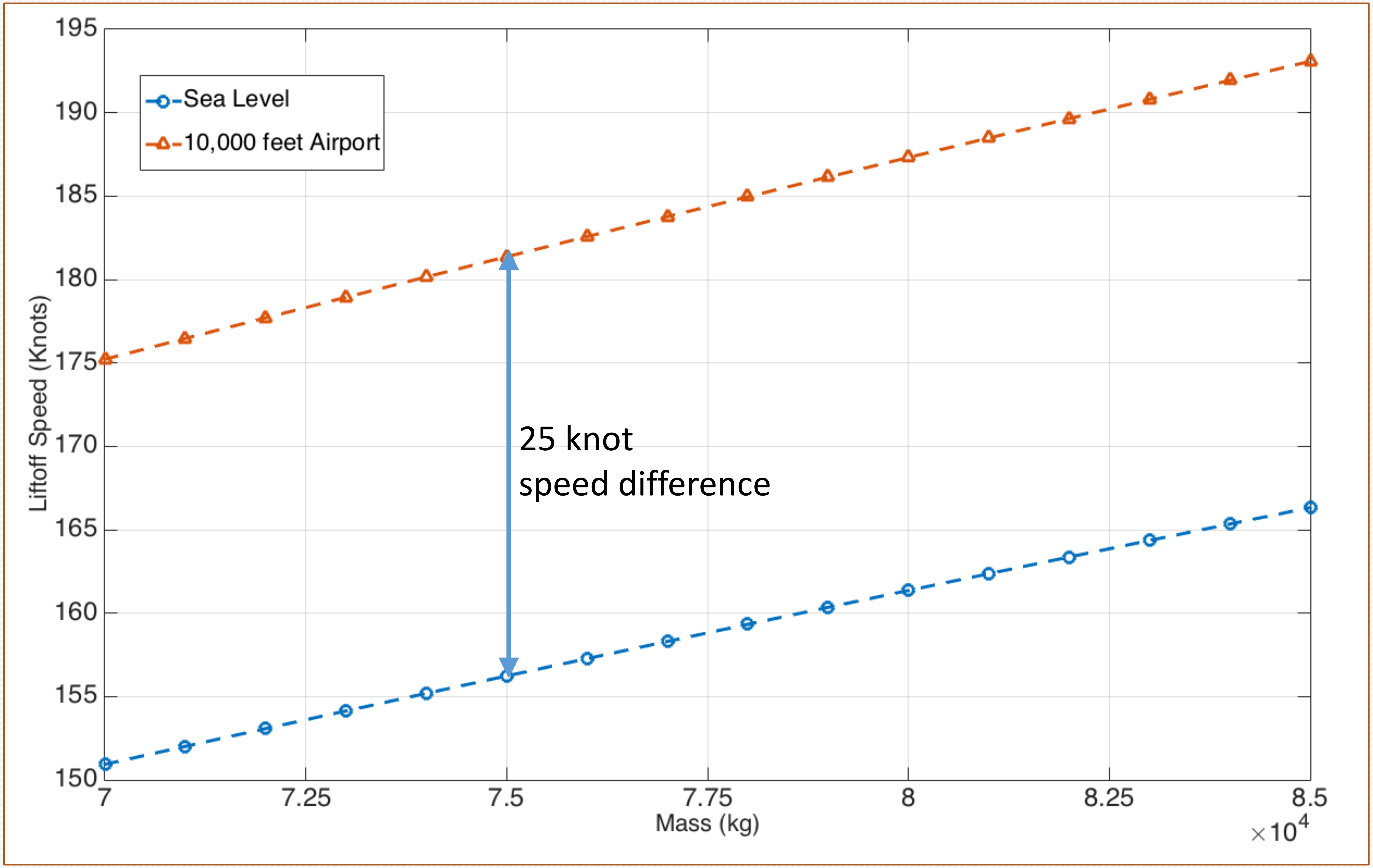
Climb speed = 1.2 V_{stall}

$$V_{liftoff} = 85.03 * 1.1 = 93.53\ m/s = 181\ knots$$

$$V_{climb} = 85.03 * 1.2 = 102.03\ m/s = 198\ knots$$

Liftoff speed increased by 19% compared to sea level conditions

Lift-Off Speeds Variations with Aircraft Mass



At high elevation airfields, the liftoff speed increases to keep the aircraft flying with the required FAR margin

Obtaining Runway Length : Integration of Equation 2.10

First obtain the aircraft speed at time *t* ,

$$V_t = \int_{V_o}^{V_{lof}} \frac{1}{m} \left(T(V, \rho) + \frac{1}{2} \rho V^2 S (C_L f_{roll} - C_D) - mg f_{roll}\right) dt \qquad (2.11)$$

Now get the distance traveled to reach the liftoff speed, S_t

$$S_t = \int_{o}^{D_{lof}} V_t \, dt \qquad (2.12)$$

Numerical Example (Light Business Jet)

$$ma = \frac{dV}{dt} = \sum F_{external}$$

$$\frac{dV}{dt} = \left(\frac{1}{m}\right)(T - D - F_f - (mg\sin\phi))$$

$$L = \frac{1}{2}\rho V^2 S C_l$$

$$D = \frac{1}{2}\rho V^2 S C_D$$

$$F_f = (mg\cos\phi - L) f_{roll}$$

$$T = f(V, \rho)$$

$m = 13900$ kilograms

$g = 9.81\ \mathrm{m/s^2}$

$S = 50.4$ square meters

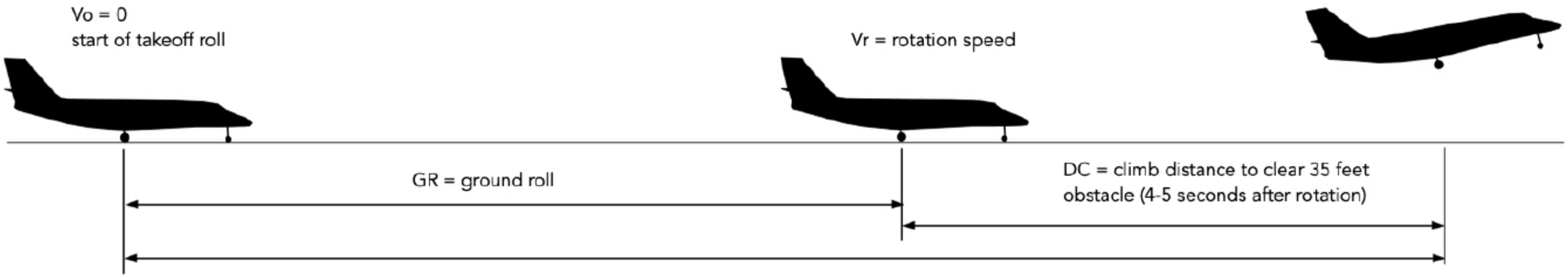
$C_D = 0.12$ (0.02 base, 0.04 flaps and 0.06 landing gear)

$C_l = 0.6$ (~zero angle of attack on takeoff roll)

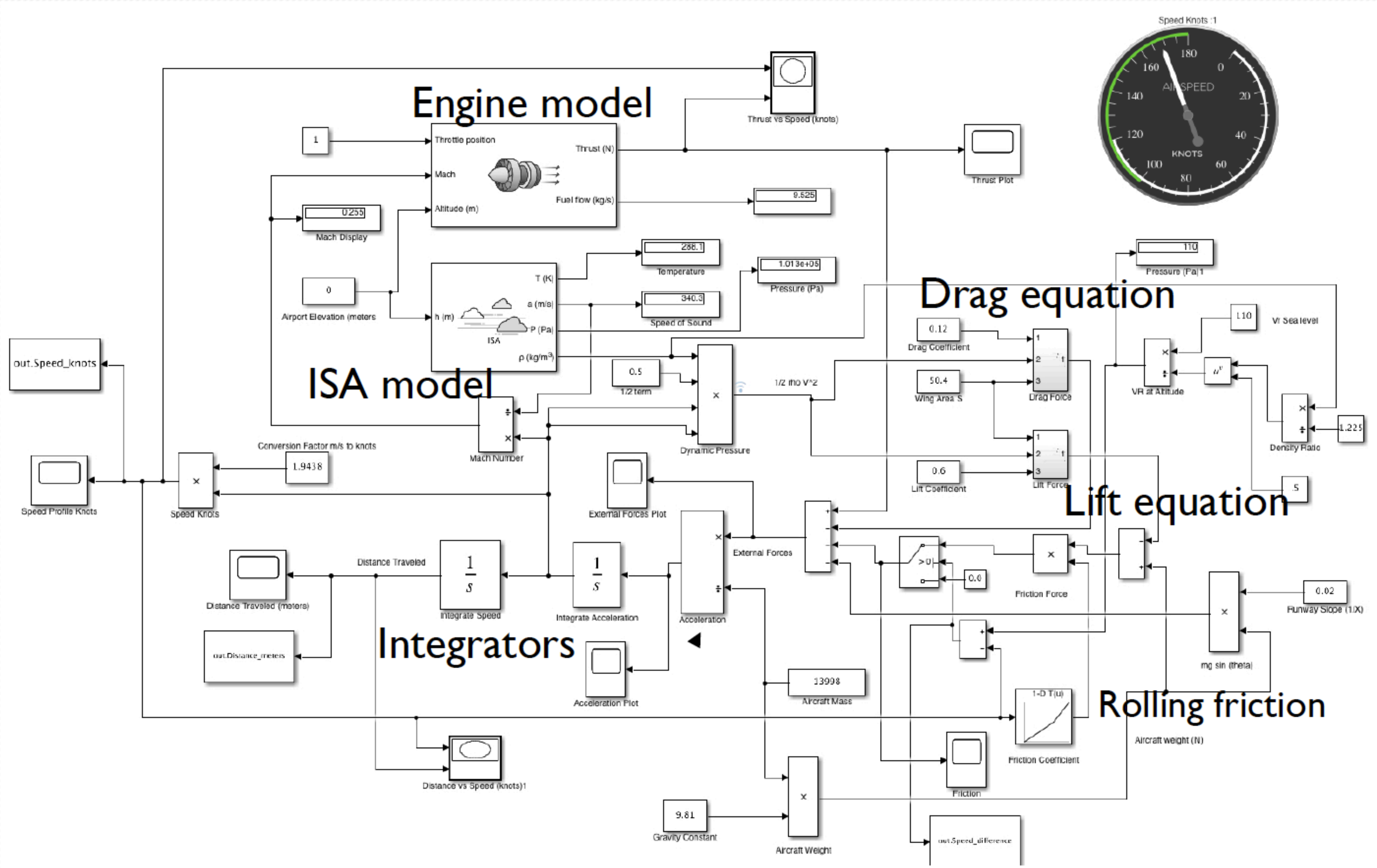
$f_{roll} = [0.025\ 0.028\ 0.030\ 0.035]$ for speed values [0 60 100 150] knots

Cessna Citation Sovereign (C680)

$$a_x = \frac{1}{m}\left(T(V,\rho) + \frac{1}{2}\rho V^2 S(C_L f_{roll} - C_D) - mgf_{roll}\right)$$



Numerical Example (Simulink Model)



Engine model

ISA model

Drag equation

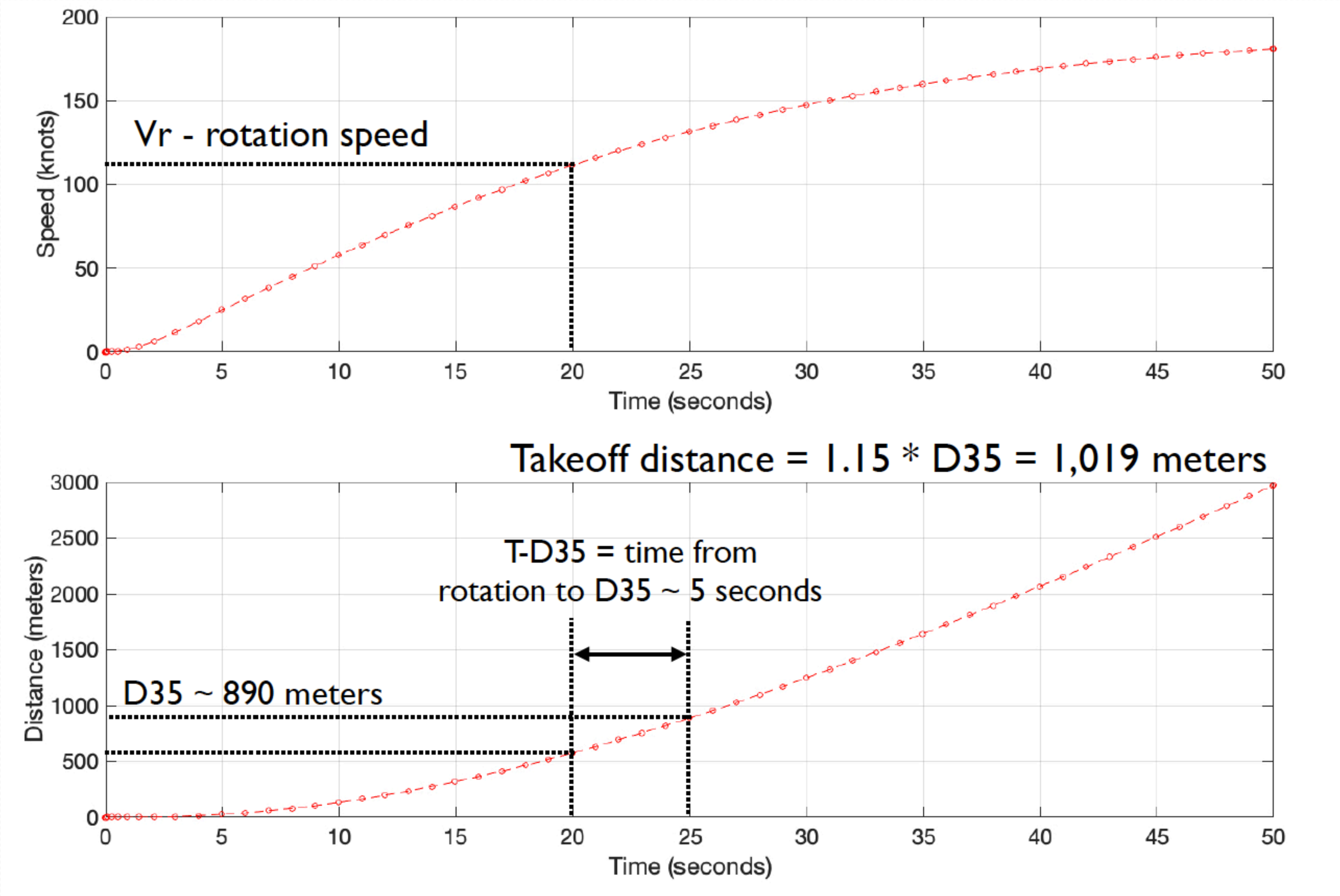
Lift equation

Integrators

Rolling friction

$$\frac{1}{m}(T(V,\rho)+\frac{1}{2}\rho V^2 S(C_L f_{roll}-C_D)-mgf_{roll}$$

Numerical Example (Simulink Model)



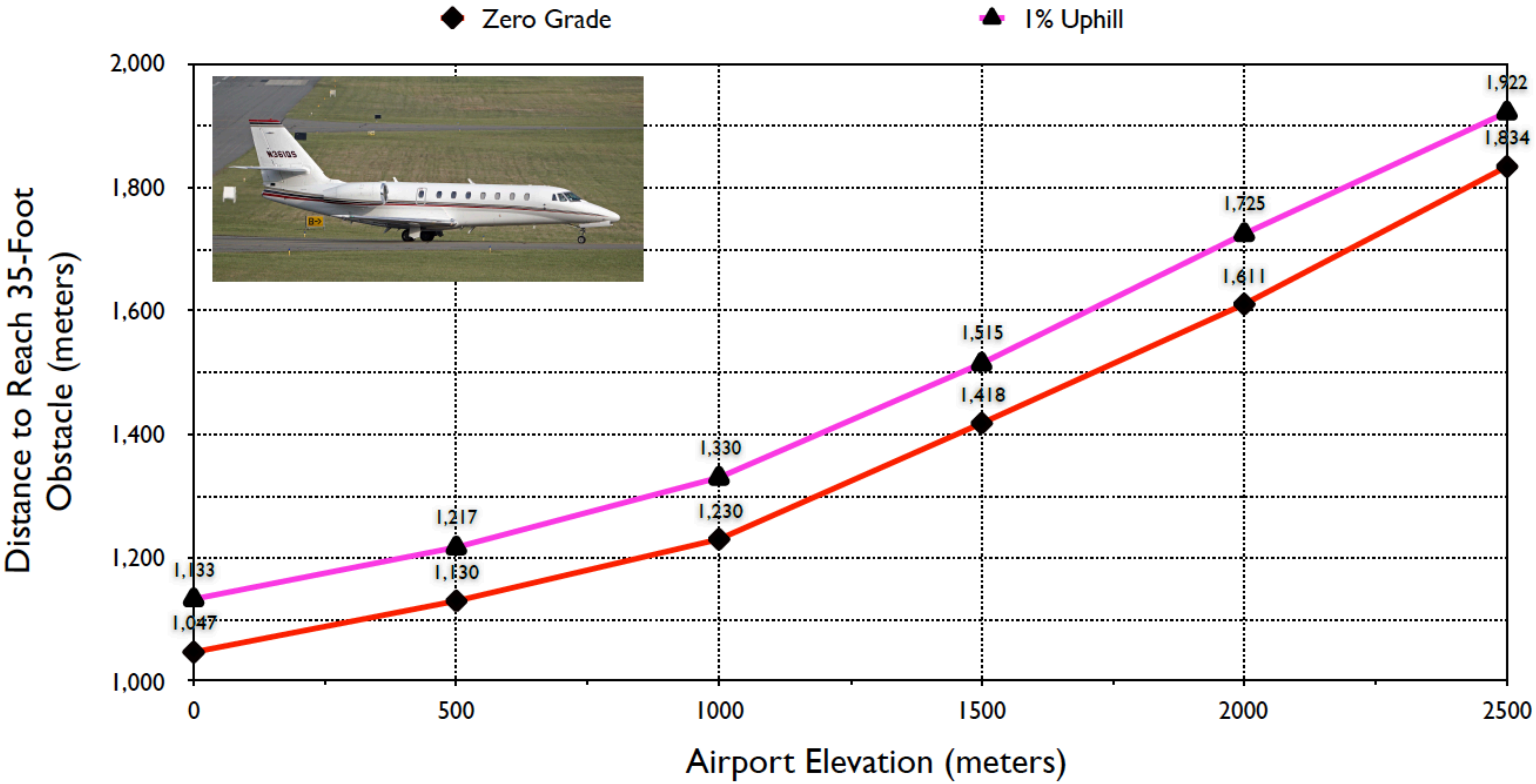
Vr - rotation speed

Takeoff distance = 1.15 * D35 = 1,019 meters

T-D35 = time from
rotation to D35 ~ 5 seconds

D35 ~ 890 meters

Numerical Results of Simulation Model



54% increase in takeoff distance at an airport 2,000 meters above mean sea level
8.2% increase in takeoff distance if the runway grade is 1% uphill

Federal Aviation Regulation Criteria to Develop Runway Length Requirements at Airports

General Procedure for Runway Length Estimation (Runway Length Components)

Runways can have three basic components:

- Full strength pavement (FS)
- Clearways (CL)
- Stopways (SW)

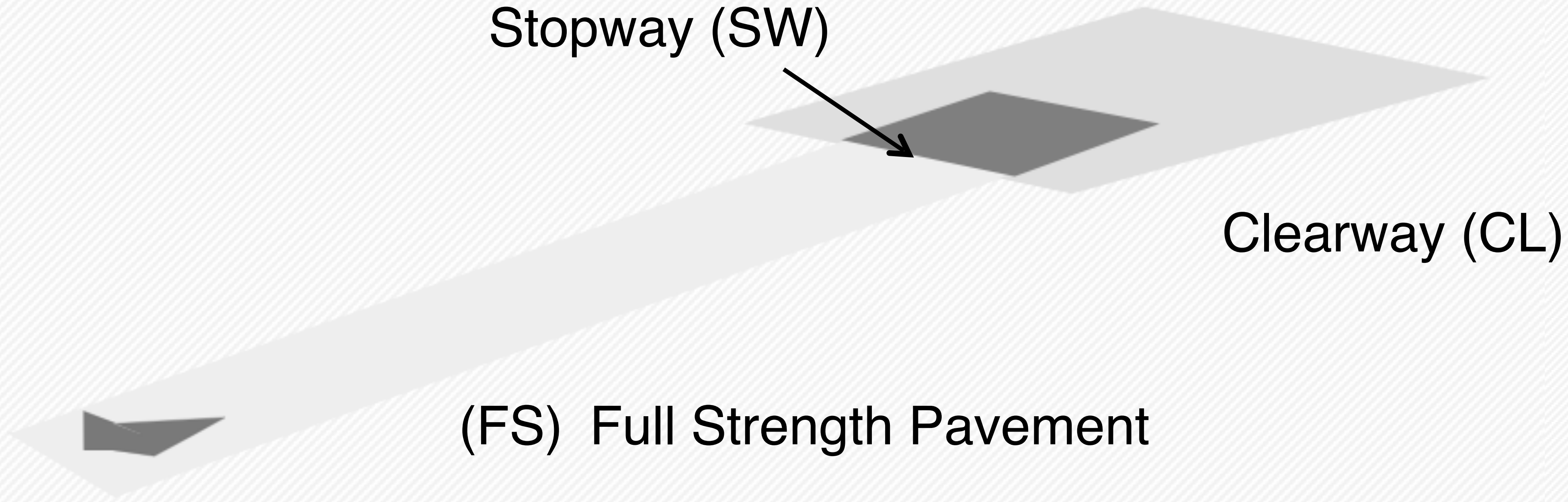
Full strength pavement should support the full weight of the aircraft.

Clearway is a prepared area are beyond FS clear of obstacles (max slope is 1.5%) allowing the aircraft to climb safely to clear an imaginary 11 m (35’ obstacle).

Stopway is a paved surface that allows and aircraft overrun to take place without harming the vehicle structurally (cannot be used for takeoff).

Runway Components

Each runway end will have to be considered individually for runway length analysis.



Stopway (SW)

Clearway (CL)

(FS) Full Strength Pavement

FAR Certification Procedures

FAR 25 (for turbojet and turbofan powered aircraft) consider three cases in the estimation of runway length performance

- Normal takeoff (all engines working fine)
- Engine-out takeoff condition
    - **Continued takeoff**
    - **Aborted takeoff**
- Landing

All these cases consider stochastic variations in piloting technique (usually very large for landings and smaller for takeoffs)

Regulations for piston aircraft do not include the normal takeoff case (an engine-out condition is more critical in piston-powered aircraft)

Runway Length Procedures (AC 150/5325-4)

Two different views of the problem:

For aircraft with MTOW up to 27,200 kg (60,000 lb.) use the aircraft grouping procedure

- **If MTOW is less than 5,670 kg use Figures 2-1 and 2-2 in FAA AC 150/5325-4**
- **If MTOW is > 5,670 kg but less than 27,200 kg use Figures 2-3 and 2-4 provided in Chapter 2 of the AC 150/5325-4**

For aircraft whose MTOW is more than 27,200 kg (60,000 lb.) use the critical aircraft concept

- **The critical aircraft is that one with the longest runway performance characteristics**
- **This aircraft needs to be operated 250 times in the year from that airport**

Review some examples

Nomenclature

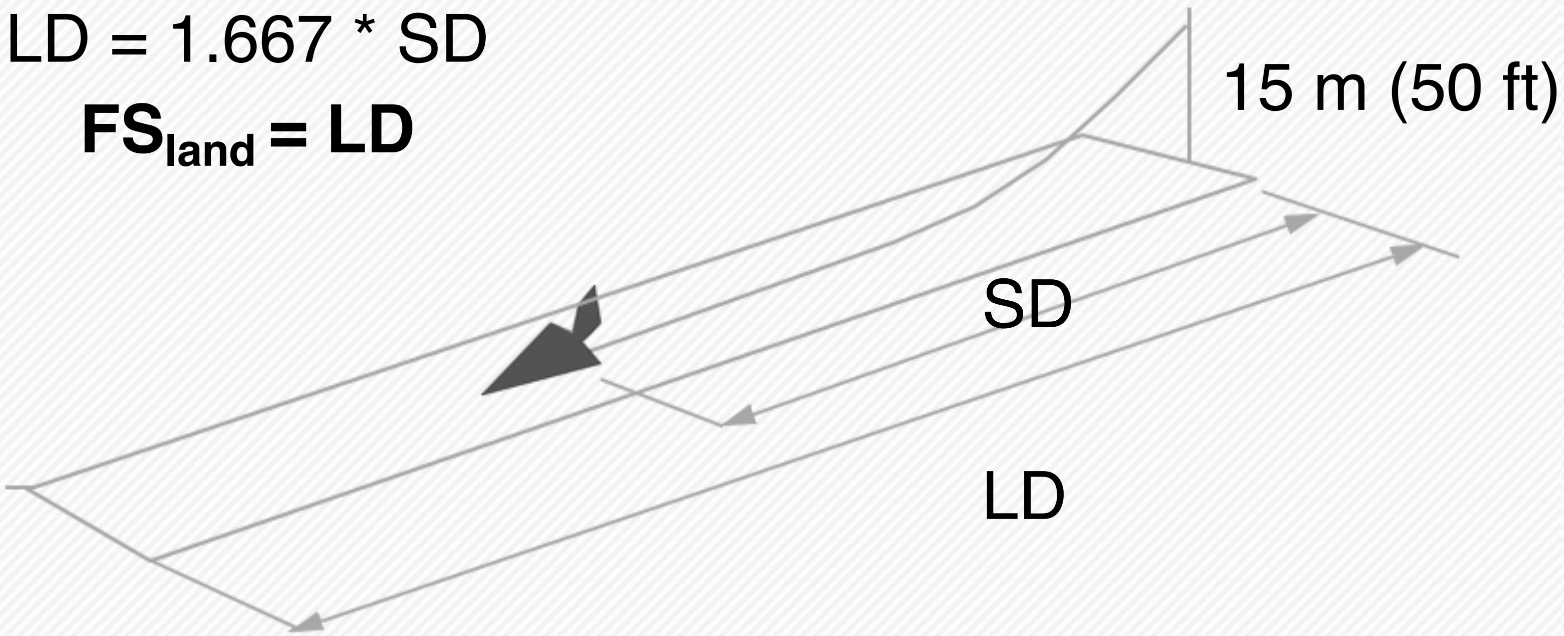
FL	field length (total amount of runway needed)
FS	field length (total amount of runway needed)
CL	clearway distance
SW	stopway distance
LOD	lift off distance
TOR	takeoff run
TOD	takeoff distance
LD	landing distance
SD	stopping distance
D35	distance to clear an 11 m (35 ft.) obstacle

Landing Distance Case

The landing distance should be 67% longer than the demonstrated distance to stop an aircraft (SD)

Large landing roll variations exist among pilots

Example touchdown point variability (μ=400 m, σ=125 m for Boeing 727-200 landing in Atlanta)

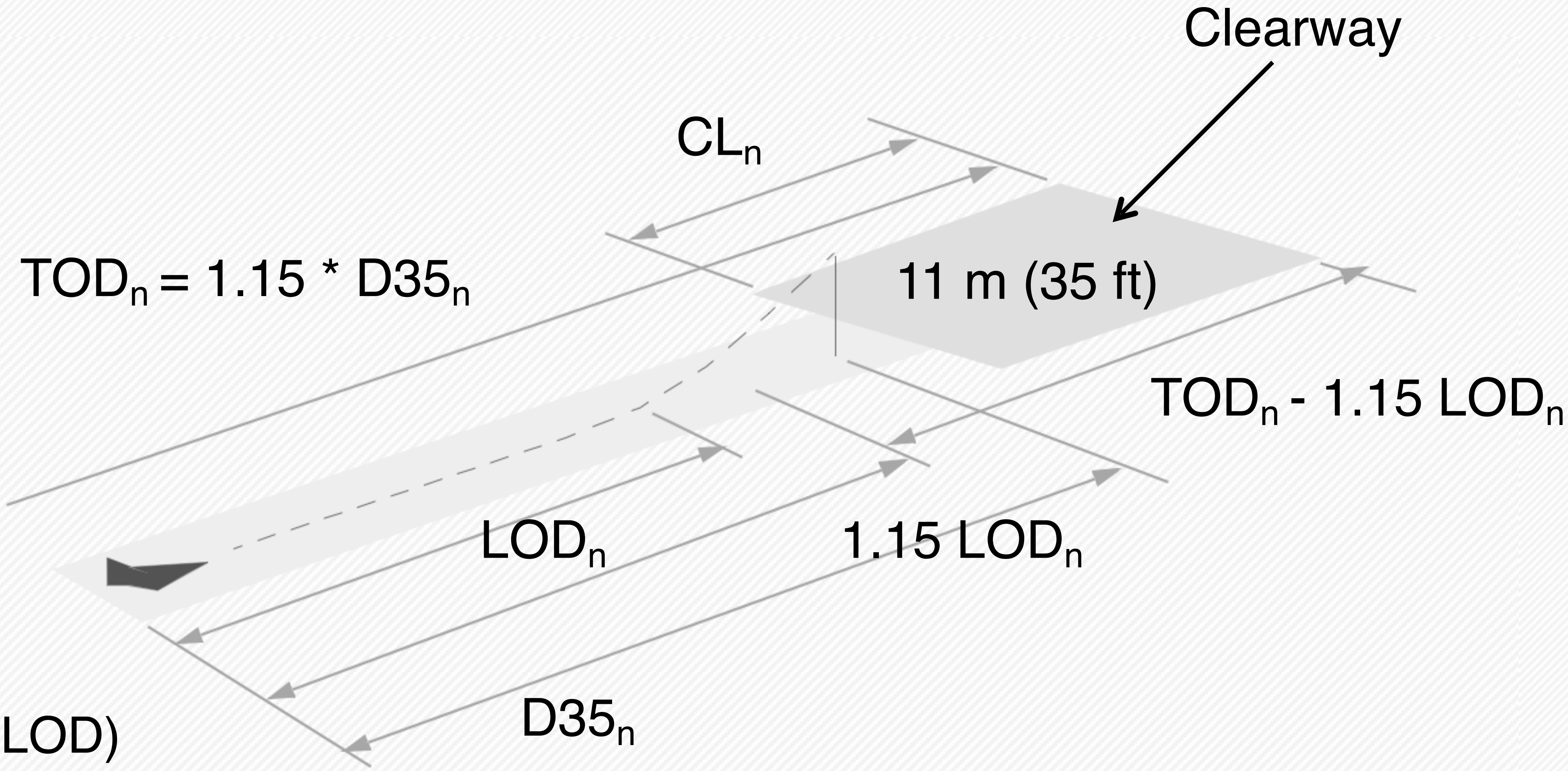


LD = 1.667 * SD

FS_{land} = LD

Normal Takeoff Case

The Takeoff Distance (TOD) should be 115% longer than the demonstrated Distance to Clear an 11m (35 ft.) obstacle (D35)



Clearway

CL_n

$TOD_n = 1.15 * D35_n$

11 m (35 ft)

$TOD_n - 1.15\ LOD_n$

LOD_n

$1.15\ LOD_n$

$D35_n$

Relationships

- $CL_n = 1/2\ (TOD-1.15\ LOD)$
- $TOR_n = TOD_n - CL_n$
- $FS_n = TOR_n$
- $FL_n = FS_n + CL_n$

Engine-Out Takeoff Case

Dictated by two scenarios:

Continued takeoff subcase

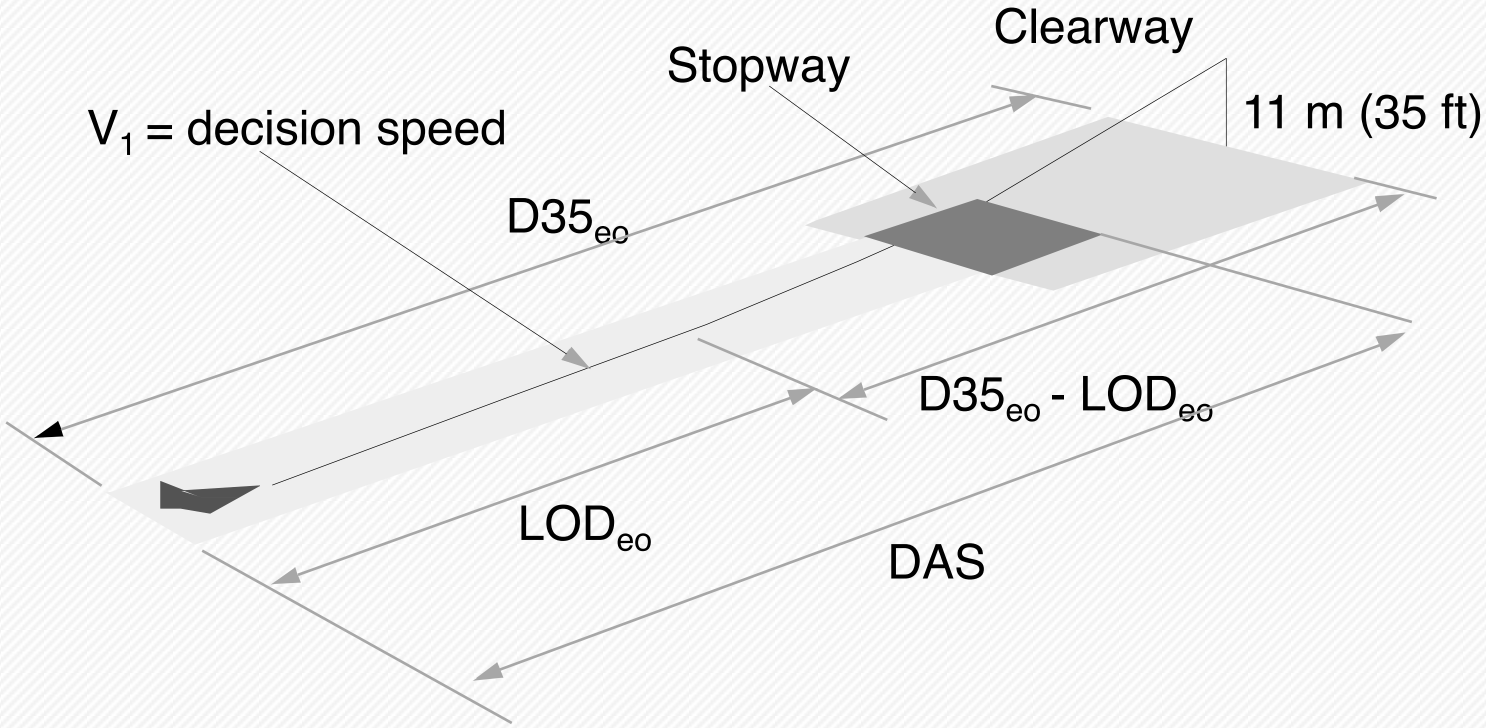
- Actual distance to clear an imaginary 11 m (35 ft) obstacle D35 (with an engine-out)

Aborted or rejected takeoff subcase

- Distance to accelerate and stop (DAS)

NOTE: no correction is applied due to the rare nature of engine-out conditions in practice for turbofan/turbojet powered aircraft

Engine-Out Analysis



Aborted Takeoff

- $FS_{eo\text{-}a}$ = DAS - SW
- $FL_{eo\text{-}a} = FS_{eo\text{-}a}$ + SW

Continued Takeoff

- $TOD_{eo} = D35_{eo}$
- $CL_{eo} = 1/2\ (D35_{eo} - LOD_{eo})$
- $TOR_{eo} = D35_{eo} - CL_{eo}$ $FS_{eo\text{-}c} = TOR_{eo}$
- $FL_{eo\text{-}c} = FS_{eo\text{-}c} + CL_{eo}$